Realizing Scale-invariant Density Perturbations in Low-energy Effective String Theory

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We discuss the realization of inflation and resulting cosmological perturbations in the low-energy effective string theory. In order to obtain nearly scale-invariant spectra of density perturbations and a suppressed tensor-to-scalar ratio, it is generally necessary that the dilaton field ϕ is effectively decoupled from gravity together with the existence of a slowly varying dilaton potential. We also study the effect of second-order corrections to the tree-level action which are the sum of a Gauss-Bonnet term coupled to ϕ and a kinetic term $(\nabla\phi)^4$. We find that it is possible to realize observationally supported spectra of scalar and tensor perturbations provided that the correction is dominated by the $(\nabla\phi)^4$ term even in the absence of the dilaton potential. When the Gauss-Bonnet term is dominant, tensor perturbations exhibit violent negative instabilities on small-scales about a de Sitter background in spite of the fact that scale-invariant scalar perturbations can be achieved.

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I. INTRODUCTION

The constantly accumulating observational data including WMAP [1, 2], SDSS [3] and 2dF [4] have continued to confirm that primordial density perturbations are adiabatic and nearly scale-invariant. This is consistent with the prediction of inflationary paradigm which is based upon a scalar field with a slowly varying potential (see Refs. [5, 6, 7] for review).

So far there are many attempts to try to explain the origin of density perturbations in the context of string theory. For example, in the low-energy effective string theory, a dilaton field ϕ coupled to a scalar curvature R leads to a so-called Pre-Big-Bang phase during which superinflation occurs in the string frame [8]. In the Einstein frame, this corresponds to a contracting universe driven by the kinetic energy of the dilaton. In this case the spectrum of curvature perturbations is highly blue-tilted ($n_{\mathcal{R}} = 4$) [9], thus incompatible with observations unless another scalar field such as axion works as a curvaton [10].

The ekpyrotic/cyclic scenarios [11] also lead to a contracting universe driven by a negative exponential potential. The perturbation spectra generated in these scenarios have been discussed by many authors (see e.g., [12]). In order for a pre-bounce growing mode of a scale-invariant Bardeen potential Φ to survive long after the bounce, it is necessary that the pressure perturbation is directly proportional to a Bardeen potential, but this mode is not present for any known form of an ordinary matter [13]. Hence it is still a challenging task to realize scale-invariant density perturbations in bouncing cosmologies.

Recently a number of authors have discussed the spectra of density perturbations in string-gas cosmology [14]. The authors in Refs. [14, 15] argued that a nearly scale-invariant perturbation may be obtained in the Hagedorn regime seeded by a gas of strings in thermal equilibrium. In recent papers it was realized that the original models in Refs. [14] lead to a highly blue-tilted spectrum ($n_{\mathcal{R}} = 5$) unless the dilaton is fixed in the Hagedorn phase [16, 17]. The coupling of the dilaton to gravity nontrivially changes the cosmological evolution and resulting density perturbations.

The above examples show that it is generally difficult to obtain nearly scale-invariant cosmological perturbations without using slow-roll inflation driven by a potential energy of a scalar field minimally coupled to gravity. In the low-energy effective string theory, the potential of the dilaton field is absent at the tree level, thus posing a difficulty to get slow-roll inflation. Moreover when the dilaton is coupled to a Ricci scalar, this has a significant effect on the spectrum of density perturbations as it happens in Pre-Big-Bang and string-gas cosmologies. It is known that in string theories there are higher-order quantum corrections, which can alter these results. However, the inflationary solutions and the resulting cosmological perturbations have not been studied much when these higher-order effects are present. It is thus important to examine what is the physical significance of these quantum corrections.

In this paper we study the possibility to achieve observationally supported cosmological perturbations in the low-energy string effective action with a dilaton coupling $F(\phi)R$. Since both curvature and tensor metric perturbations are invariant under a conformal transformation in the absence of higher-order corrections, the demand for realizing scale-invariant spectra in the string frame and a suppressed tensor-to-scalar ratio corresponds to obtaining slow-roll inflation in the Einstein frame. This then requires the presence of a dilaton potential together with the condition that the dilaton is effectively decou-

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pled from gravity when the perturbations on cosmologically relevant scales are generated.

We also take into account the higher-order corrections which are the sum of the contributions of a Gauss-Bonnet (GB) term and a kinetic term $(\nabla\phi)^4$. When the $(\nabla\phi)^4$ term is dominant, we find that it is in fact possible to realize observationally supported density perturbations even if the dilaton potential is absent. The reason why this "kinetic inflation" [18] can be successful is that inflation of a slow-roll type is realized in the Einstein frame, unlike the Pre-Big-Bang and ekpyrotic/cyclic models.

Inflation can be also realized when the GB term is present. However this does not necessarily mean that the models are compatible with observations. In fact, with several different choices of the coupling $F(\phi)$, we show that either of the following two cases occurs when the GB term is dominant:

(i) The power spectra of curvature and tensor perturbations are far from scale-invariant ones, or

(ii) While the spectra of scalar perturbations can be nearly scale-invariant, tensor perturbations exhibit negative instabilities on small scales which invalidates the assumption of linear perturbations.

Thus the GB-dominated inflation is generally problematic to generate observationally supported density perturbations from quantum fluctuations. However, we find that if the GB term is subdominant compared with the higher-order kinetic term, it is possible to obtain the density perturbations compatible with observations.

This paper is organized as follows. In Sec. II the formulae for the spectra of scalar and tensor metric perturbations are presented for the low-energy string effective action with higher-order correction terms. In Sec. III we discuss the transformation of the action to the Einstein frame and show the equivalence of perturbation spectra in both the string and Einstein frames. In Sec. IV we study the possibility to obtain scale-invariant cosmological perturbations in dilaton gravity without higher-order corrections. We consider general models without restricting the Pre-Big-Bang cosmology in the latter half of Sec. IV. We then investigate cosmological perturbations in the theories with higher-order kinetic terms $(\nabla\phi)^4$ in Sec. V. Models with higher-order corrections which are the sum of the GB term and the $(\nabla\phi)^4$ term are discussed in Sec. VI. Our new findings are the results presented in Sec. V and Sec. VI together with the latter half of Sec. IV. Sec. VII is devoted to conclusions.

II. COSMOLOGICAL PERTURBATIONS FOR A GENERAL ACTION

In this section, we first develop the formalism for analysing the density perturbations for a system with higher-order corrections. Specifically we consider the following general action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(\phi)R - \frac{1}{2}\omega(\phi)(\nabla\phi)^2 - V(\phi) + \mathcal{L}_c\right], \qquad (1)$$

where ϕ is a scalar field with a potential $V(\phi)$, R is a Ricci scalar, $F(\phi)$ and $\omega(\phi)$ are the functions of ϕ. In what follows, we use the unit $\kappa^2 \equiv 8\pi G = 1$ with G being gravitational constant. The Lagrangian $\mathcal{L}_c$ represents the contribution of higher-order corrections to be specified shortly.

In the weak coupling limit of the low-energy effective string theory, where the string coupling $g_s^2 \equiv e^{\phi}$ is much smaller than unity, the action corresponds to $F(\phi) = e^{-\phi}$, $\omega(\phi) = -e^{-\phi}$, and $V(\phi) = 0$ [8, 19]. We only consider a dilaton field ϕ by assuming that other modulus fields corresponding to the size of extra dimensions are fixed through some mechanism.[1] The higher-order corrections $\mathcal{L}_c$ are the infinite sums of the series expansion with an expansion parameter $\alpha' = l_s^2$, where l_s is the string length. We pick up lowest-order terms with which the equations of motion are second-order in fields [21, 22]:

$$\mathcal{L}_c = -\frac{1}{2}\alpha'\xi(\phi)\left[c_1 R_{\rm GB}^2 + c_2(\nabla\phi)^4\right], \qquad (2)$$

where $R_{\rm GB}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$ is the GB term. At the tree level, the coupling $\xi(\phi)$ and the coefficients c_1 and c_2 are

$$\xi(\phi) = \lambda e^{-\phi}, \quad c_1 = 1, \quad c_2 = -1, \qquad (3)$$

where $\lambda = -1/4, -1/8, 0$ correspond to bosonic, heterotic and Type II superstrings, respectively. In what follows, we work in a unit $\alpha' = 1$.

As the system enters a large coupling region characterized by $g_s^2 \gtrsim 1$, it is expected that the forms of the functions $F(\phi)$, $\omega(\phi)$ and $\xi(\phi)$ become more complicated than those given above. Moreover the potential of the dilaton may appear in order to stabilize the field. Hence we work on the general action (1) without restricting ourselves to the tree-level case.

In a flat Friedmann-Robertson-Walker (FRW) metric with a scale factor a, we obtain the following background equations [23, 24, 25]:

$$H^2 = \frac{1}{6F}\left(\omega\dot{\phi}^2 + 2V - 6H\dot{F} + 2\rho_c\right), \qquad (4)$$

$$\dot{H} = \frac{1}{2F}\left(-\omega\dot{\phi}^2 + H\dot{F} - \ddot{F} - \rho_c - p_c\right), \qquad (5)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{1}{2\omega}\left(\omega_{,\phi}\dot{\phi}^2 - F_{,\phi}R + 2V_{,\phi} + T_c\right) = 0, \qquad (6)$$

[1] See Refs. [20] for the the cosmological evolution in the presence of modulus fields.

where a dot represents the time derivative, $\omega_{,\phi} \equiv \mathrm{d}\omega/\mathrm{d}\phi$, $H \equiv \dot{a}/a$, and the correction terms are

$$\rho_c = 12c_1\dot{\xi}H^3 - \frac{3}{2}c_2\xi\dot{\phi}^4, \tag{7}$$

$$p_c = -4c_1[\ddot{\xi}H^2 + 2\dot{\xi}H(\dot{H}+H^2)] - \frac{1}{2}c_2\xi\dot{\phi}^4, \tag{8}$$

$$T_c = 24c_1\xi_{,\phi}(\dot{H}+H^2)H^2 \\ -c_2\dot{\phi}^2(3\dot{\xi}\dot{\phi} + 12\xi\ddot{\phi} + 12\xi\dot{\phi}H). \tag{9}$$

Let us consider the following perturbed metric about a FRW background [26]:

$$\mathrm{d}s^2 = -(1+2A)\mathrm{d}t^2 + 2a\partial_i B \mathrm{d}x^i\mathrm{d}t \\ +a^2\left[(1+2\psi)\delta_{ij} + 2\partial_{ij}E + 2h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\,, \tag{10}$$

where ∂_i represents the spatial partial derivative $\partial/\partial x^i$ and $\partial_{ij} = \nabla_i\nabla_j - (1/3)\delta_{ij}\nabla^2$. We use lower case latin indices running over the 3 spatial coordinates. We note that A, B, ψ and E denote scalar metric perturbations, whereas h_{ij} represents tensor perturbations. We define the so-called comoving perturbation [27]

$$\mathcal{R} \equiv \psi - \frac{H}{\dot{\phi}}\delta\phi\,, \tag{11}$$

which is invariant under a gauge transformation. The Fourier modes of curvature perturbations satisfy [23, 24]

$$\frac{1}{az^2}(az^2\dot{\mathcal{R}})^{\cdot} + c_{\mathcal{R}}^2\frac{k^2}{a^2}\mathcal{R} = 0\,, \tag{12}$$

where k is a comoving wavenumber and

$$z^2 = \frac{a^2\left(\omega\dot{\phi}^2 + \frac{3(\dot{F}+Q_1)^2}{2F+Q_2} + Q_3\right)}{\left(H + \frac{\dot{F}+Q_1}{2F+Q_2}\right)^2}, \tag{13}$$

$$c_{\mathcal{R}}^2 = 1 + \frac{Q_4 + \frac{\dot{F}+Q_1}{2F+Q_2}Q_5 + \left(\frac{\dot{F}+Q_1}{2F+Q_2}\right)^2 Q_6}{\omega\dot{\phi}^2 + \frac{3(\dot{F}+Q_1)^2}{2F+Q_2} + Q_3}, \tag{14}$$

with

$$Q_1 = -4c_1\dot{\xi}H^2, \quad Q_2 = -8c_1\dot{\xi}H, \quad Q_3 = -6c_2\xi\dot{\phi}^4, \\ Q_4 = 4c_2\xi\dot{\phi}^4, \quad Q_5 = -16c_1\dot{\xi}\dot{H}, \quad Q_6 = 8c_1(\ddot{\xi} - \dot{\xi}H). \tag{15}$$

Introducing a new variable, $v = z\mathcal{R}$, we find that Eq. (12) can be rewritten as

$$v'' + \left(c_{\mathcal{R}}^2k^2 - \frac{z''}{z}\right)v = 0\,, \tag{16}$$

where a prime represents a derivative with respect to a conformal time $\tau = \int a^{-1}\mathrm{d}t$. When the evolution of z is given by $z \propto |\tau|^q$, one has $z''/z = \gamma_{\mathcal{R}}/\tau^2$ with $\gamma_{\mathcal{R}} = q(q-1)$. In this case, if $c_{\mathcal{R}}^2$ is a positive constant, the solution for Eq. (16) can be written by using Hankel functions:

$$v = \frac{\sqrt{\pi|\tau|}}{2}\left[c_1(k)H^{(1)}_{\nu_{\mathcal{R}}}(c_{\mathcal{R}}k|\tau|) + c_2(k)H^{(2)}_{\nu_{\mathcal{R}}}(c_{\mathcal{R}}k|\tau|)\right], \tag{17}$$

where $\nu_{\mathcal{R}} = \sqrt{\gamma_{\mathcal{R}} + 1/4} = |q - 1/2|$. We choose the coefficients to be $c_1 = 0$ and $c_2 = 1$, so that positive frequency solutions in a Minkowski vacuum are recovered in an asymptotic past. Since $H^{(2)}_{\nu_{\mathcal{R}}}(c_{\mathcal{R}}k|\tau|) \to (i/\pi)\Gamma(\nu_{\mathcal{R}})(c_{\mathcal{R}}k|\tau|/2)^{-\nu_{\mathcal{R}}}$ for long wavelength perturbations ($c_{\mathcal{R}}k|\tau| \ll 1$), the curvature perturbation after the Hubble radius crossing is given by

$$\mathcal{R} = i\frac{\sqrt{|\tau|}}{4z}\frac{\Gamma(\nu_{\mathcal{R}})}{\Gamma(3/2)}\left(\frac{c_{\mathcal{R}}k|\tau|}{2}\right)^{-\nu_{\mathcal{R}}}. \tag{18}$$

The spectrum of the curvature perturbation is defined by $\mathcal{P}_{\mathcal{R}} = k^3|\mathcal{R}|^2/2\pi^2$. Then we find

$$\mathcal{P}_{\mathcal{R}} = \frac{c_{\mathcal{R}}^{-2\nu_{\mathcal{R}}}}{Q_{\mathcal{R}}}\left(\frac{H}{2\pi}\right)^2\left(\frac{1}{aH|\tau|}\right)^2\left(\frac{\Gamma(\nu_{\mathcal{R}})}{\Gamma(3/2)}\right)^2\left(\frac{k|\tau|}{2}\right)^{3-2\nu_{\mathcal{R}}} \\ \equiv A_{\mathcal{R}}^2\left(\frac{k|\tau|}{2}\right)^{3-2\nu_{\mathcal{R}}}, \tag{19}$$

where

$$Q_{\mathcal{R}} \equiv \frac{\omega\dot{\phi}^2 + \frac{3(\dot{F}+Q_1)^2}{2F+Q_2} + Q_3}{\left(H + \frac{\dot{F}+Q_1}{2F+Q_2}\right)^2}. \tag{20}$$

When $\nu_{\mathcal{R}} = 0$, we have an additional $\ln(k|\tau|)$ factor. From Eq. (19) the spectral index of the power spectrum is

$$n_{\mathcal{R}} - 1 = 3 - 2\nu_{\mathcal{R}} = 3 - \sqrt{4\gamma_{\mathcal{R}} + 1}\,. \tag{21}$$

Note that the scale-invariant spectrum ($n_{\mathcal{R}} = 1$) corresponds to $\gamma_{\mathcal{R}} = 2$ (or $\nu_{\mathcal{R}} = 3/2$).

We decompose tensor perturbations into eigenmodes of the spatial Lagrangian, $\nabla^2 e_{ij} = -k^2 e_{ij}$, with scalar amplitude $h(t)$, i.e., $h_{ij} = h(t)e_{ij}$, where e_{ij} have two polarization states. The Fourier modes of tensor perturbations satisfy [23, 24]

$$\frac{1}{a^3Q_T}(a^3Q_T\dot{h})^{\cdot} + c_T^2\frac{k^2}{a^2}h = 0\,, \tag{22}$$

where

$$Q_T = F + \frac{Q_2}{2}\,, \quad c_T^2 = 1 - \frac{4c_1(\ddot{\xi} - \dot{\xi}H)}{F - 4c_1\dot{\xi}H}\,. \tag{23}$$

Introducing new variables $z_T = a\sqrt{Q_T}$ and $v_T = z_T h/2$, Eq. (22) can be rewritten as

$$v_T'' + \left(c_T^2k^2 - \frac{z_T''}{z_T}\right)v_T = 0\,. \tag{24}$$

The power spectrum of tensor perturbations is defined by $\mathcal{P}_T = 2\,k^3|h|^2/2\pi^2$ because of two polarization states of the graviton. If c_T^2 is a positive constant and the evolution of z_T is given by $z_T \propto |\tau|^{q_T}$, we obtain

$$\begin{aligned}\mathcal{P}_T &= \frac{8c_T^{-2\nu_T}}{Q_T}\left(\frac{H}{2\pi}\right)^2\left(\frac{1}{aH|\tau|}\right)^2\left(\frac{\Gamma(\nu_T)}{\Gamma(3/2)}\right)^2\left(\frac{k|\tau|}{2}\right)^{3-2\nu_T} \\ &\equiv A_T^2\left(\frac{k|\tau|}{2}\right)^{3-2\nu_T},\end{aligned} \tag{25}$$

where $\nu_T = \sqrt{\gamma_T + 1/4}$ with $\gamma_T = q_T(q_T-1)$. The spectral index of the power spectrum is

$$n_T = 3 - 2\nu_T = 3 - \sqrt{4\gamma_T + 1}\,. \tag{26}$$

The tensor-to-scalar ratio is given by

$$\begin{aligned} r &\equiv \frac{A_T^2}{A_{\mathcal{R}}^2} \\ &= 8\frac{\omega\dot{\phi}^2 + \frac{3(\dot{F}+Q_1)^2}{2F+Q_2} + Q_3}{\left(H + \frac{\dot{F}+Q_1}{2F+Q_2}\right)^2\left(F + \frac{Q_2}{2}\right)}\frac{c_{\mathcal{R}}^{2\nu_{\mathcal{R}}}}{c_T^{2\nu_T}}\left(\frac{\Gamma(\nu_T)}{\Gamma(\nu_{\mathcal{R}})}\right)^2.\end{aligned} \tag{27}$$

We introduce the following quantities

$$\begin{aligned}&\epsilon_1 = -\frac{\dot{H}}{H^2}\,, \quad \epsilon_2 = \frac{\ddot{\phi}}{H\dot{\phi}}\,, \quad \epsilon_3 = \frac{\dot{F}}{2HF}\,, \quad \epsilon_4 = \frac{\dot{E}}{2HE}\,, \\ &\epsilon_5 = \frac{\dot{F}+Q_1}{H(2F+Q_2)}\,, \quad \epsilon_6 = \frac{\dot{Q}_T}{2HQ_T}\,,\end{aligned} \tag{28}$$

where

$$E \equiv \frac{F}{\dot{\phi}^2}\left[\omega\dot{\phi}^2 + \frac{3(\dot{F}+Q_1)^2}{2F+Q_2} + Q_3\right]. \tag{29}$$

Then the variable z^2 in Eq. (13) is given by

$$z^2 = \left(\frac{a\dot{\phi}}{H(1+\epsilon_5)}\right)^2\frac{E}{F}\,. \tag{30}$$

In what follows, we specialize to the case in which $\dot{\epsilon}_i$ and $\ddot{\epsilon}_i$ terms vanish or the case in which they can be neglected compared to other terms (like slow-roll inflation). Since the conformal time is given by $\tau = -1/(aH(1-\epsilon_1))$ in this case, we get $z''/z = \gamma_{\mathcal{R}}/\tau^2$ with

$$\gamma_{\mathcal{R}} = \frac{(1+\epsilon_1+\epsilon_2-\epsilon_3+\epsilon_4)(2+\epsilon_2-\epsilon_3+\epsilon_4)}{(1-\epsilon_1)^2}\,. \tag{31}$$

Then the spectral index of scalar perturbations is given by

$$n_{\mathcal{R}} - 1 = 3 - \left|\frac{3+\epsilon_1+2\epsilon_2-2\epsilon_3+2\epsilon_4}{1-\epsilon_1}\right|\,. \tag{32}$$

The variable z_T for tensor perturbations satisfies the relation $z_T''/z_T = \gamma_T/\tau^2$ with

$$\gamma_T = \frac{(2-\epsilon_1+\epsilon_6)(1+\epsilon_6)}{(1-\epsilon_1)^2}\,. \tag{33}$$

Hence the spectral index of tensor perturbations is

$$n_T = 3 - \left|\frac{3-\epsilon_1+2\epsilon_6}{1-\epsilon_1}\right|\,. \tag{34}$$

When the conditions $|\epsilon_i| \ll 1$ hold, the above spectral indices are approximately given by

$$n_{\mathcal{R}} - 1 = -2(2\epsilon_1+\epsilon_2-\epsilon_3+\epsilon_4)\,, \tag{35}$$

$$n_T = -2(\epsilon_1+\epsilon_6)\,. \tag{36}$$

Since $\nu_{\mathcal{R}} \simeq 3/2 \simeq \nu_T$ in this case, the tensor-to-scalar ratio (27) reads

$$r = 8\frac{\omega\dot{\phi}^2 + \frac{3(\dot{F}+Q_1)^2}{2F+Q_2} + Q_3}{\left(H + \frac{\dot{F}+Q_1}{2F+Q_2}\right)^2\left(F + \frac{Q_2}{2}\right)}\left(\frac{c_{\mathcal{R}}}{c_T}\right)^3. \tag{37}$$

Let us consider the case in which the GB term is absent ($c_1 = 0$). Since $Q_T = F$ and $\epsilon_6 = \epsilon_3$, one has $n_T = -2(\epsilon_1+\epsilon_3)$. From Eq. (5) we obtain the relation $\omega\dot{\phi}^2/H^2F - 2c_2\xi\dot{\phi}^4/H^2F \simeq 2(\epsilon_1+\epsilon_3)$. Then with the use of Eq. (14) the tensor-to-scalar ratio (37) is simply given by $r = 16(\epsilon_1+\epsilon_3)c_{\mathcal{R}}$. Hence we find

$$r = -8c_{\mathcal{R}}n_T \quad (\text{for} \ \ c_1 = 0). \tag{38}$$

When the $c_2(\nabla\phi)^4$ term is absent, one has $c_{\mathcal{R}} = 1$, thereby reducing to the standard consistency relation: $r = -8n_T$ [7]. Provided that $\mathcal{L}_c = 0$, this standard consistency relation holds even for scalar-tensor models characterized by the action (1) [24, 28].

III. EINSTEIN FRAME

The action (1) is transformed to the one in the Einstein frame by a conformal transformation [29]:

$$\hat{g}_{\mu\nu} = \Omega g_{\mu\nu}\,, \quad \Omega = F\,, \tag{39}$$

where a hat represents quantities in the Einstein frame. For later convenience, we write the correction term $\mathcal{L}_c$ as $\mathcal{L}_c = \mathcal{L}_{\rm GB} - (1/2)c_2\xi(\phi)(\nabla\phi)^4$, where $\mathcal{L}_{\rm GB}$ is the contribution of the GB term. Then the action in the Einstein frame is

$$\begin{aligned}S_E = \int d^4\hat{x}\sqrt{-\hat{g}}\biggl[&\frac{\hat{R}}{2} + K(\phi)X + L(\phi)X^2 \\ &-\hat{V}(\phi) + \hat{\mathcal{L}}_{\rm GB}\biggr],\end{aligned} \tag{40}$$

where $X = -(1/2)(\hat{\nabla}\phi)^2 = (1/2)(\mathrm{d}\phi/\mathrm{d}\hat{t})^2$ and

$$K(\phi) = \frac{3}{2}\left(\frac{F_{,\phi}}{F}\right)^2 + \frac{\omega}{F}\,, \tag{41}$$
$$L(\phi) = -2c_2\xi(\phi)\,, \tag{42}$$
$$\hat{V}(\phi) = \frac{V(\phi)}{F^2}\,. \tag{43}$$

Let us introduce a perturbed metric in the Einstein frame:

$$\begin{aligned}\mathrm{d}\hat{s}^2 &= \Omega \mathrm{d}s^2 \\ &= -(1+2\hat{A})\mathrm{d}\hat{t}^2 + 2\hat{a}\partial_i\hat{B}\mathrm{d}\hat{x}^i\mathrm{d}\hat{t} \\ &\quad +\hat{a}^2\left[(1+2\hat{\psi})\delta_{ij} + 2\partial_{ij}\hat{E} + 2\hat{h}_{ij}\right]\mathrm{d}\hat{x}^i\mathrm{d}\hat{x}^j\,. \end{aligned} \tag{44}$$

We decompose the conformal factor into the background and the perturbed part as

$$\Omega(\mathbf{x},t) = \bar{\Omega}(t)\left(1 + \frac{\delta\Omega(\mathbf{x},t)}{\bar{\Omega}(t)}\right)\,. \tag{45}$$

Then the following relations are derived:

$$\begin{aligned}&\hat{a} = a\sqrt{\Omega},\quad \mathrm{d}\hat{t} = \sqrt{\Omega}\mathrm{d}t,\quad \hat{H} = \frac{1}{\sqrt{\Omega}}\left(H + \frac{\dot{\Omega}}{2\Omega}\right), \\ &\hat{A} = A + \frac{\delta\Omega}{2\Omega},\quad \hat{\psi} = \psi + \frac{\delta\Omega}{2\Omega},\end{aligned} \tag{46}$$

where a "bar" is dropped from the expression of $\bar{\Omega}(t)$.

From these relations, one can show that the curvature perturbation in the Einstein frame coincides with that in the Jordan frame:

$$\begin{aligned}\hat{\mathcal{R}} &\equiv \hat{\psi} - \frac{\hat{H}}{\mathrm{d}\hat{\phi}/\mathrm{d}\hat{t}}\delta\hat{\phi} \\ &= \psi - \frac{H}{\dot{\phi}}\delta\phi = \mathcal{R}\,.\end{aligned} \tag{47}$$

Since tensor perturbations are also invariant under a conformal transformation, the power spectra of scalar and tensor perturbations satisfy

$$\hat{\mathcal{P}}_{\mathcal{R}} = \mathcal{P}_{\mathcal{R}}\,, \qquad \hat{\mathcal{P}}_T = \mathcal{P}_T\,. \tag{48}$$

We also note that the comoving wavenumber, k^2, is invariant under a conformal transformation (since in the Fourier space $\Delta = -k^2$ is invariant). Hence the amplitudes and the spectral indices of power spectra based upon $\mathcal{R}$ and h_{ij} in the string frame coincide with those in the Einstein frame.

A. Models without the GB term ($c_1 = 0$)

In this section, we investigate a situation in which the contribution of the GB term is absent [$\hat{\mathcal{L}}_{\mathrm{GB}} = 0$ in Eq. (40)]. In order to derive $n_{\mathcal{R}}$ and other physical quantities for the action (40) without the GB term, it is sufficient to use the formula obtained in the previous section by replacing $F(\phi) \to 1$, $\omega(\phi) \to K(\phi)$, $c_1 \to 0$ and $c_2\xi(\phi) \to -(1/2)L(\phi)$. Introducing the following parameters

$$\hat{\epsilon}_1 = -\frac{\mathrm{d}\hat{H}/\mathrm{d}\hat{t}}{\hat{H}^2}\,,\quad \hat{\epsilon}_2 = \frac{\mathrm{d}^2\phi/\mathrm{d}\hat{t}^2}{\hat{H}(\mathrm{d}\phi/\mathrm{d}\hat{t})}\,,\quad \hat{\epsilon}_4 = \frac{\mathrm{d}\hat{E}/\mathrm{d}\hat{t}}{2\hat{H}\hat{E}}\,, \tag{49}$$

where $E = K + 6LX$, the spectral index $n_{\mathcal{R}}$ of curvature perturbations is

$$n_{\mathcal{R}} - 1 = 3 - \left|\frac{3 + \hat{\epsilon}_1 + 2\hat{\epsilon}_2 + 2\hat{\epsilon}_4}{1 - \hat{\epsilon}_1}\right| \tag{50}$$
$$\simeq -2(2\hat{\epsilon}_1 + \hat{\epsilon}_2 + \hat{\epsilon}_4)\,. \tag{51}$$

The first equality is valid under the condition that the derivative terms of ϵ_i vanish or they are negligible compared to other terms, whereas the second approximate equality is valid under the slow-roll approximation. Similarly the spectral index of tensor perturbations is

$$n_T = 3 - \left|\frac{3 - \hat{\epsilon}_1}{1 - \hat{\epsilon}_1}\right| \tag{52}$$
$$\simeq -2\hat{\epsilon}_1\,. \tag{53}$$

When $|\epsilon_i| \ll 1$ the tensor-to-scalar ratio r satisfies the consistency relation (38) with $c_{\mathcal{R}}$ given by [30]

$$c_{\mathcal{R}}^2 = \frac{K + 2LX}{K + 6LX}\,. \tag{54}$$

B. Models without the correction term ($c_1 = c_2 = 0$)

Let us next consider the case in which the correction term $\mathcal{L}_c$ is absent ($c_1 = c_2 = 0$). When $K(\phi)$ is positive, we introduce a new scalar field φ as

$$\varphi = \int \sqrt{K(\phi)}\,\mathrm{d}\phi\,. \tag{55}$$

Then the action (40) can be written in a canonical form

$$S_E = \int \mathrm{d}^4\hat{x}\sqrt{-\hat{g}}\left[\frac{\hat{R}}{2} - \frac{1}{2}(\hat{\nabla}\varphi)^2 - \hat{V}(\varphi(\phi))\right]\,. \tag{56}$$

If $K(\phi)$ is negative, we just need to define $\varphi = \int\sqrt{-K(\phi)}\,\mathrm{d}\phi$ with the change of sign of the kinetic term in Eq. (56).

Introducing the parameters

$$\hat{\epsilon}_1 = -\frac{\mathrm{d}\hat{H}/\mathrm{d}\hat{t}}{\hat{H}^2}\,,\quad \hat{\epsilon}_2 = \frac{\mathrm{d}^2\varphi/\mathrm{d}\hat{t}^2}{\hat{H}(\mathrm{d}\varphi/\mathrm{d}\hat{t})}\,, \tag{57}$$

the spectral indices $n_{\mathcal{R}}$ and n_T are given by Eqs. (50)-(53) with $\hat{\epsilon}_4 = 0$. From Eq. (27), the tensor-to-scalar ratio r is

$$r = 16\hat{\epsilon}_1\left(\frac{\Gamma(\hat{\nu}_T)}{\Gamma(\hat{\nu}_{\mathcal{R}})}\right)^2\,, \tag{58}$$

where we have used $\mathrm{d}\hat{H}/\mathrm{d}\hat{t} = -(\mathrm{d}\varphi/\mathrm{d}\hat{t})^2/2$. Note that this relation is obtained without using the slow-roll approximation. In order to satisfy the condition $r \ll 1$, which is supported from observations, we should have $\hat{\epsilon}_1 \ll 1$ provided that $\Gamma(\hat{\nu}_T)/\Gamma(\hat{\nu}_{\mathcal{R}})$ is of order 1. The condition to obtain a nearly scale-invariant scalar perturbation ($n_{\mathcal{R}} \simeq 1$) then gives two cases: (i) $\hat{\epsilon}_2 \simeq 0$ or (ii) $\hat{\epsilon}_2 \simeq -3$. The former corresponds to the standard slow-roll inflation. The latter corresponds to a constant scalar-field potential ($\hat{V}_{,\varphi} = 0$). In both cases, we have $r \simeq 16\epsilon_1$ since $\nu_{\mathcal{R}} \simeq \nu_T \simeq 3/2$.

One may define standard slow-roll parameters in terms of the slope of the potential

$$\hat{\epsilon} = \frac{1}{2}\left(\frac{\hat{V}_{,\varphi}}{\hat{V}}\right)^2 , \quad \hat{\eta} = \frac{\hat{V}_{,\varphi\varphi}}{\hat{V}} . \tag{59}$$

Under a slow-roll approximation ($|\epsilon_i| \ll 1$) one has $\hat{\epsilon}_1 \simeq \hat{\epsilon}$ and $\hat{\epsilon}_2 \simeq \hat{\epsilon} - \hat{\eta}$. Hence we get the following usual formulae [6, 7]:

$$n_{\mathcal{R}} - 1 \simeq -6\hat{\epsilon} + 2\hat{\eta} , \quad n_T \simeq -2\hat{\epsilon} , \quad r \simeq 16\hat{\epsilon} . \tag{60}$$

IV. DILATON GRAVITY WITHOUT THE CORRECTION TERM ($\mathcal{L}_c = 0$)

Let us first consider dilaton gravity without the correction term $\mathcal{L}_c$ and study the possibility to obtain scale-invariant cosmological perturbations. The low-energy string effective action corresponds to $F(\phi) = e^{-\phi}$, $\omega(\phi) = -e^{-\phi}$, $V(\phi) = 0$ at the tree level. In the Einstein frame the system is described by a minimally coupled field φ without a potential, thus giving the following solution [8]:

$$\hat{a} \propto |\hat{t}|^{1/3} , \quad \hat{H} = \frac{1}{3\hat{t}} , \quad \left(\frac{\mathrm{d}\varphi}{\mathrm{d}\hat{t}}\right)^2 = \frac{2}{3\hat{t}^2} . \tag{61}$$

When $\hat{t} < 0$, this solution corresponds to a collapsing universe. In the Pre-Big-Bang cosmology, the universe contracts in the Einstein frame for $\hat{t} < 0$, which is followed by the bounce around $\hat{t} = 0$ because of the effect of higher-order loop and derivative corrections [22, 31]. Note that a causal mechanism for perturbations works in the collapsing universe as in the case of standard inflation. If we consider a growing field φ for $\hat{t} < 0$, we have $\mathrm{d}\varphi/\mathrm{d}\hat{t} = -\sqrt{2/3}(1/\hat{t})$.

We find from Eq. (61) that $\hat{\epsilon}_1$ and $\hat{\epsilon}_2$ defined in Eq. (49) are constants, i.e.,

$$\hat{\epsilon}_1 = 3 , \quad \hat{\epsilon}_2 = -3 . \tag{62}$$

In this case one can use the formula (50), (52) and (58) with $\hat{\epsilon}_4 = 0$, which gives

$$n_{\mathcal{R}} = 4 , \quad n_T = 3 , \quad r = 48 . \tag{63}$$

This is a highly blue-tilted spectrum and a large tensor-to-scalar ratio incompatible with observations. Although these perturbations correspond to the ones which are generated before the bounce, it was shown that these spectra are preserved even long after the bounce if α' curvature and derivative corrections are taken into account [32].

The reason why the spectrum is highly blue-tilted is that the system is dominated by the kinetic energy of the scalar field. In order to obtain nearly scale-invariant spectra, we should require that $\hat{\epsilon}_1$ and $\hat{\epsilon}_2$ are much smaller than 1. This situation is not realized unless a slowly varying potential is present in the Einstein frame. In order to investigate the possibility of obtaining nearly scale-invariant perturbation spectra, in what follows, we study models with general forms of $F(\phi)$ and $\omega(\phi)$ together with the dilaton potential $V(\phi)$. Note that the results given in Eq. (63) for the Pre-Big-Bang model were already derived in past works (e.g. Ref. [9]), but the results we present below in more general models are new.

It follows from Eq. (58) that we must have $\hat{\epsilon} \ll 1$ to realize the condition $r \ll 1$ unless the $(\Gamma(\hat{\nu}_T)/\Gamma(\hat{\nu}_{\mathcal{R}}))^2$ term is much smaller than unity. Let us clarify whether or not this condition can be satisfied in dilaton gravity. For the coupling $F(\phi) = e^{-\phi}$ and $\omega(\phi) = -e^{-\phi}$, the dilaton potential in the Einstein frame is given by

$$\hat{V} = e^{2\phi}V(\phi) = e^{2\sqrt{2}\varphi}V(\sqrt{2}\varphi) , \tag{64}$$

where we have used Eqs. (43) and (55). Since $\varphi = \phi/\sqrt{2}$, the parameter $\hat{\epsilon}$ defined in Eq. (59) is given by

$$\hat{\epsilon} = \left(2 + \frac{V_{,\phi}}{V}\right)^2 . \tag{65}$$

Even when a slowly varying potential is present in the string frame ($|V_{,\phi}/V| \ll 1$), we get $\hat{\epsilon} \simeq 4$, thus leading to a large tensor-to-scalar ratio. The potential which gives $\hat{\epsilon} = 0$ is characterized by

$$V(\phi) = V_0 e^{-2\phi} , \tag{66}$$

where V_0 is a constant. In fact in this case the potential in the Einstein frame is exactly constant, thus giving $n_{\mathcal{R}} = 1$ and $r = 0$ because $\hat{\epsilon}_1 = 0$ and $\hat{\epsilon}_2 = -3$. Hence for the coupling $F(\phi) = e^{-\phi}$ and $\omega(\phi) = -e^{-\phi}$, it is required that the dilaton potential in the string frame (approximately) takes the form (66) to realize $n_{\mathcal{R}} \simeq 1$ and $r \ll 1$. We note however that the potential of the dilaton is absent in the perturbative regime we considered above.

Let us consider more general functions of $F(\phi)$ and $\omega(\phi)$ with a dilaton potential $V(\phi)$. Then the slow-roll parameter $\hat{\epsilon}$ is given by

$$\hat{\epsilon} = \frac{1}{3F_{,\phi}^2/F^2 + 2\omega/F}\left(\frac{V_{,\phi}}{V} - 2\frac{F_{,\phi}}{F}\right)^2 . \tag{67}$$

This shows that there are three cases in which the condition $\hat{\epsilon} \ll 1$ is satisfied:
(i) $V_{,\phi}/V \simeq 2F_{,\phi}/F$,
(ii) $1 \gg |V_{,\phi}/V| \gg |2F_{,\phi}/F|$,
(iii) $1 \gg |2F_{,\phi}/F| \gg |V_{,\phi}/V|$.

Note that we are considering the situation where the $|\omega/F|$ term is of order unity. In a special case with $|\omega/F| \gg 1$, the condition $\hat{\epsilon} \ll 1$ can be satisfied even for $|\omega/F| \gg (V_{,\phi}/V)^2 \gg (F_{,\phi}/F)^2 \gg 1$. One may also think that the slow-roll condition can be fulfilled even for $V = 0$ provided $|F_{,\phi}/F| \ll 1$, but the definition of $\hat{\epsilon}$ itself is not meaningful in this case because of the absence of the potential.

The case (i) is equivalent to the condition

$$V(\phi) \propto F^2(\phi)\,, \tag{68}$$

which, in the Einstein frame, corresponds to a cosmological constant from Eq. (43).

The cases (ii) and (iii) are difficult to realize if the dilaton coupling $F(\phi)$ changes rapidly as in the tree-level action. For the coupling $F(\phi) \sim e^{\lambda\phi}$ we require the condition $|\lambda| \ll 1$, but it is generally difficult to take a control of the rapid evolution of the dilaton in the non-perturbative regime. However one may consider so-called a runaway dilaton scenario [33] in which the field is gradually decoupled from gravity in the regime $e^{\phi} \gg 1$ (after the field passes the non-perturbative region). In this scenario the functions $F(\phi)$ and $\omega(\phi)$ are assumed to take the following forms for $e^{\phi} \gg 1$:

$$F(\phi) = C_1 + D_1 e^{-\phi} + \mathcal{O}(e^{-2\phi})\,, \tag{69}$$

$$\omega(\phi) = C_2 + D_2 e^{-\phi} + \mathcal{O}(e^{-2\phi})\,. \tag{70}$$

Let us study the case in which the potential of the dilaton is present in this region. Then the slow-roll parameter $\hat{\epsilon}$ is approximately given by

$$\hat{\epsilon} \simeq \frac{C_1}{2C_2}\left(\frac{V_{,\phi}}{V} + 2\frac{D_1}{C_1}e^{-\phi}\right)^2\,. \tag{71}$$

In the cases (ii) and (iii) we have $|V_{,\phi}/V| \gg |2(D_1/C_1)e^{-\phi}|$ and $|V_{,\phi}/V| \ll |2(D_1/C_1)e^{-\phi}|$, respectively. One may consider a potential in which the field has a minimum at $\phi = \phi_0$, i.e., $V(\phi) = \lambda_n(\phi - \phi_0)^n$. When $1 \gg |V_{,\phi}/V| \gg |2(D_1/C_1)e^{-\phi}|$, we obtain $n_{\mathcal{R}}$ and r as in the case of standard chaotic inflation [7]. In other words, it is necessary that the dilaton is effectively decoupled from gravity in order to realize standard slow-roll inflation in the Einstein frame. Note that inflation is also realized for $|V_{,\phi}/V| \ll |2(D_1/C_1)e^{-\phi}| \ll 1$ except for the case in which the potential of the dilaton vanishes.

The above discussion shows how the coupling of dilaton to gravity alters the spectra of density perturbations. Except for the specific case given in Eq. (68), the dilaton needs to be almost decoupled from gravity before perturbations on cosmologically relevant scales are generated.

V. KINETIC INFLATION ($c_1 = 0, c_2 \neq 0, V = 0$)

When $c_1 = 0$ and $c_2 \neq 0$, it is known that kinetic inflation can be realized in the Einstein frame even in the absence of the dilaton potential [18]. This corresponds to the action (40) with $\hat{V} = 0$ and $\hat{\mathcal{L}}_{\rm GB} = 0$. Then the background equations in the Einstein frame are given by

$$3\hat{H}^2 = \hat{\rho}\,, \tag{72}$$

$$2\dot{\hat{H}} = -(\hat{\rho} + \hat{p})\,, \tag{73}$$

where

$$\hat{p} = K(\phi)X + L(\phi)X^2\,, \tag{74}$$

$$\hat{\rho} = K(\phi)X + 3L(\phi)X^2\,. \tag{75}$$

In the weak coupling regime characterized by $e^{\phi} \ll 1$, one has $F(\phi) = e^{-\phi}$, $\omega(\phi) = -e^{-\phi}$, $\xi(\phi) = \lambda e^{-\phi}$ and $c_2 = -1$. Hence the functions $K(\phi)$ and $L(\phi)$ are given by $K(\phi) = 1/2$ and $L(\phi) = 2\lambda e^{-\phi}$. Since $\lambda = -1/4, -1/8$ for bosonic and heterotic strings, respectively, the function $L(\phi)$ is negative at the tree level. In the strong coupling regime ($e^{\phi} \gtrsim 1$), the forms of the functions $K(\phi)$ and $L(\phi)$ are expected to be modified by higher-order quantum effects.

From the above equations, we find that a de Sitter solution ($\dot{\hat{H}} = 0$) is present when the condition

$$K + 2LX = 0\,, \tag{76}$$

is satisfied. In this case one gets $3\hat{H}^2 = K^2/(4L)$, thereby requiring $L > 0$. Then from Eq. (76), the function K is negative for the existence of the de Sitter solution. In what follows, we concentrate on the case where $K < 0$ and $L > 0$. Note that ghost condensate model proposed in Ref. [34] corresponds to $K = -1$ and $L = 1$.

While $K(\phi)$ is positive at the tree level, it can change sign in the strongly coupled regime. In fact taking into account two-derivative perturbative loop corrections to the Kahler potential derived from heterotic string theory compactified on a Z_N orbifold, the function $\omega(\phi)$ is subject to change [31]

$$\omega(\phi) = -e^{-\phi}\left[1 + \frac{3}{2}\frac{be^{\phi}(6 + be^{\phi})}{(3 + be^{\phi})^2}\right]\,, \tag{77}$$

where b is a positive constant of order unity. Then from Eq. (41) with $F(\phi) = e^{-\phi}$, the function $K(\phi)$ is given by

$$K(\phi) = \frac{1}{2} - \frac{3}{2}\frac{be^{\phi}(6 + be^{\phi})}{(3 + be^{\phi})^2}\,, \tag{78}$$

which is negative for $be^{\phi} > 3(\sqrt{6} - 2)/2$. This function has the asymptotic behaviors $K(\phi) \simeq -1 + 27e^{-2\phi}/2b^2 \to -1$ for $e^{\phi} \gg 1$ and $K(\phi) \simeq 1/2 - be^{\phi} \to 1/2$ for $e^{\phi} \ll 1$. Note that the above corrections are obtained perturbatively, so they may not be valid in a full non-perturbative regime. But this example is instructive to show the possibility to obtain negative values of $K(\phi)$ after the system enters the strongly coupled region.

When $K(\phi)$ is negative, it is possible to realize kinetic inflation provided that $L(\phi) > 0$. At the tree level, $L(\phi)$ is negative for bosonic and heterotic strings, but it can be

positive in the strongly coupled region. One may write $\xi(\phi)$ in terms of the sums of the tree-level and n-loop corrections in the form

$$\xi(\phi) = \sum_{n=0} C_n e^{(n-1)\phi}\,, \tag{79}$$

where $n = 0$ corresponds to the tree-level term with $C_0 = 1$. Unfortunately all the coefficients C_n have not been determined so far. So in what follows, we discuss the theory with the couplings given in (79) with various C_n. This is at least useful to clarify the condition under which kinetic inflation generates nearly scale-invariant density perturbations.

It follows from Eq. (54) that $c_{\mathcal{R}}^2 \simeq 0$ along the de Sitter point (76) and $c_T^2 = 1$ from Eq. (23). By using the formula given in Eqs. (51), (53) and (38), we obtain

$$n_{\mathcal{R}} - 1 = -2\Bigg[\frac{6(K+2LX)}{K+3LX} + \frac{\dot{X}}{2\hat{H}X} + \frac{(K_{,\phi}+6XL_{,\phi})\dot{\phi}+6L\dot{X}}{2\hat{H}(K+6LX)}\Bigg]\,, \tag{80}$$

$$n_T = -\frac{6(K+2LX)}{K+3LX}\,, \tag{81}$$

$$r = -8\sqrt{\frac{K+2LX}{K+6LX}}\,n_T\,. \tag{82}$$

Then along the de Sitter fixed point ($X \simeq -K/2L$), one has

$$n_T \simeq 0\,, \quad r \simeq 0\,. \tag{83}$$

Meanwhile the spectrum of curvature perturbations is given by

$$n_{\mathcal{R}} - 1 = \left(-\frac{2K_{,\phi}}{\hat{H}K} + \frac{L_{,\phi}}{\hat{H}L}\right)\dot{\phi}\,. \tag{84}$$

By using $\hat{H} = \sqrt{-K/12}|\dot{\phi}|$ along the de Sitter solution, this is written as

$$n_{\mathcal{R}} - 1 = \frac{\mathrm{sign}(\dot{\phi})}{\sqrt{-K/12}}\left(-\frac{2K_{,\phi}}{K} + \frac{L_{,\phi}}{L}\right)\,. \tag{85}$$

Let us first investigate the case in which $L(\phi)$ is a constant. We are interested in the evolution of dilaton from a strongly coupled region ($e^\phi \gg 1$ and $K < 0$) to a weakly coupled region ($e^\phi \ll 1$ and $K > 0$), which means that $\dot{\phi} < 0$ and $K_{,\phi} < 0$. Alternatively it is possible to consider the case with $\dot{\phi} > 0$ and $K_{,\phi} > 0$ in which the system gradually decouples from gravity in the region $e^\phi \gg 1$ as in the runaway dilaton scenario. In both cases, we obtain a blue-tilted spectrum $n_{\mathcal{R}} > 1$.

We next study a situation in which one of the terms in Eq. (79), say $e^{\lambda\phi}$, dominates over other terms in the strongly coupled region (as in the dilatonic ghost condensate model proposed in Ref. [35]). This then gives

$$n_{\mathcal{R}} - 1 = \frac{\mathrm{sign}(\dot{\phi})}{\sqrt{-K/12}}\left(\lambda - \frac{2K_{,\phi}}{K}\right)\,. \tag{86}$$

If the terms higher than the one-loop corrections are dominant, i.e., $\lambda \geq 1$, the power spectrum strongly departs from the scale-invariant one. Thus in order to be compatible with observations, we must require that the function $\xi(\phi)$ be nearly constant in the regime $e^\phi \gg 1$. Taking the terms up to the first order in Eq. (79), we have $L_{,\phi}/L \simeq -e^{-\phi}/C_1 > 0$ (because C_1 is negative for the positivity of L). If $L_{,\phi}/L$ is larger than $2K_{,\phi}/K$, the red-tilted spectrum follows for $\dot{\phi} < 0$. For example, if the function $K(\phi)$ is given by Eq. (78), we find

$$n_{\mathcal{R}} - 1 \simeq \frac{\mathrm{sign}(\dot{\phi})}{\sqrt{-K/12}}\left(-\frac{54}{b^2}e^{-2\phi} + \frac{1}{|C_1|}e^{-\phi}\right)\,, \tag{87}$$

where the second term in the square bracket dominates over the first one. Hence in this case one gets a red-tilted spectrum for $\dot{\phi} < 0$. Of course if the function $K(\phi)$ takes a different form such as $K(\phi) = -\tanh(\lambda\phi)$, it can happen that the spectrum is blue-tilted.

Thus the spectral index of curvature perturbations depends upon the forms of $K(\phi)$ and $L(\phi)$ in the case of de Sitter solutions. Let us estimate $n_{\mathcal{R}}$ on the cosmologically relevant scales observed in Cosmic Microwave Background anisotropies. For simplicity, we choose the function $K(\phi)$ given in Eq. (78) with constant $L(\phi)$. Along the de Sitter solution (76) one has $\dot{\phi} = \sigma\sqrt{-K/L}$ and $\hat{H} = -K/2\sqrt{3L}$ where $\sigma = \pm 1$. This then gives the number of e-foldings:

$$N \equiv \int_t^{t_f} \hat{H}\mathrm{d}\hat{t} = \frac{\sigma}{2\sqrt{3}}\int_\phi^{\phi_f}\sqrt{-K}\mathrm{d}\phi\,, \tag{88}$$

where the subscript "f" represents the values at the end of inflation. If inflation occurs in the region $be^\phi \gg 1$, we get $N \simeq (\phi - \phi_f)/2\sqrt{3}$ (here negative σ is chosen).

Considering homogeneous perturbations δX around X [18], we find that $p_{,X} = 2L\delta X$ and $\dot{\rho} = -2\sqrt{3\rho}Xp_{,X}$. Then one has $\delta X/X = -(\sigma/\sqrt{3})(-K)_{,\phi}/(-K)^{3/2}$. Inflation ends when the $\delta X/X$ term grows of order unity. Taking the criterion $(-K)_{,\phi}/(-K)^{3/2} = \sqrt{3}$ for the end of inflation, we obtain $be^{\phi_f} \sim 1.75$ for the model (78). This gives $be^\phi = 1.75e^{2\sqrt{3}N}$. Then we finally get the spectral index in terms of the function of N:

$$n_{\mathcal{R}} - 1 = 4\sqrt{3}\frac{(-K)_{,\phi}}{(-K)^{3/2}} \simeq 60e^{-4\sqrt{3}N}\,. \tag{89}$$

For the cosmologically relevant scales ($N \sim 60$), the spectrum is extremely close to scale-invariant one. This situation does not change provided that $K(\phi)$ and $L(\phi)$ are nearly constants with an exponentially suppressed factor. These models, which give $n_{\mathcal{R}} \simeq 1$ and $r \simeq 0$, are interesting to confront with future high-precision observations, since recent WMAP3 data do not favor an exact scale-invariant scalar perturbation with a vanishing tensor-to-scalar ratio [2].

We note that there are some cases in which the spectrum is tilted from the scale-invariant one. As an example, let us consider the realization of power-law kinetic

inflation which is characterized by

$$\hat{a} \propto \hat{t}^{1/\gamma}\,, \quad \hat{H} = \frac{1}{\gamma \hat{t}}\,, \tag{90}$$

where $0 < \gamma < 1$. For the choice $L(\phi) = -K(\phi)$, we find from Eqs. (72) and (73) that

$$X = \frac{3-\gamma}{3(2-\gamma)}\,, \quad K(\phi) = -\frac{6(2-\gamma)}{\gamma^2(\phi-\phi_0)^2}\,, \tag{91}$$

where ϕ_0 is the initial value of the field. Hence from Eqs. (50), (52) and (27) with $\hat{\epsilon}_1 = -\hat{\epsilon}_4 = \gamma$, $\hat{\epsilon}_2 = 0$ and $c_{\mathcal{R}}^2 = \gamma/(3(4-\gamma)) > 0$, we obtain

$$n_{\mathcal{R}} - 1 = n_T = -\frac{2\gamma}{1-\gamma}\,, \tag{92}$$

$$r = 16\gamma \left[\frac{\gamma}{3(4-\gamma)}\right]^{\frac{1+\gamma}{2(1-\gamma)}}\,. \tag{93}$$

This indicates that the spectra of scalar and tensor perturbations are red-tilted ($n_{\mathcal{R}} < 1$ and $n_T < 0$). The parameter γ is a measure of the departure from the Harrison-Zeldvich scale-invariant spectra ($n_{\mathcal{R}} - 1 = n_T = 0$ and $r = 0$). The recent WMAP3 data [2] give the constraints $n_{\mathcal{R}} = 0.987^{+0.019}_{-0.037}$ and $r < 0.55$ at the 2σ level for the ΛCDM model without the running of scalar perturbations. The constraint (92) then requires that γ should be $\gamma < 0.024$. This in turn gives $r < 0.0148$ which is much smaller than 0.55. This shows that the constraint from $n_{\mathcal{R}}$ gives a severer bound on γ. Finally we note that in order to end the power-law inflation the functions $K(\phi)$ and $L(\phi)$ need to be modified in a suitable way.

VI. MODELS WITH THE CORRECTION TERM

In this section we investigate models in which the correction term $\mathcal{L}_c$ is present. We are interested in the realization of inflation without using a dilaton potential.[2] Basically one may consider the following two situations: (i) $F(\phi)$ asymptotically approaches a constant value (as in the runaway dilaton scenario), or (ii) $F(\phi)$ is the sum of exponential terms. In the case (i), we set $F(\phi) = 1$ by assuming that the time-derivatives of $F(\phi)$ in Eqs. (4) and (5) are negligible relative to others. In the case (ii), we study models in which one of the exponential terms dominates over others, i.e., $F(\phi) = e^{\mu\phi}$.

A. Models with $F(\phi) \sim$ constant

In order to understand the effect of the GB term to realize inflation, we first study the case $c_2 = 0$ and $F(\phi) = 1$. Then Eqs. (4) and (5) yield

$$6H^2 = \omega\dot{\phi}^2 + 24c_1\dot{\xi}H^3\,, \tag{94}$$

$$2\dot{H} = -\omega\dot{\phi}^2 + 4c_1\left[H^2\ddot{\xi} + (2H\dot{H} - H^3)\dot{\xi}\right]. \tag{95}$$

We search for power-law solutions given by

$$a \propto t^{1/\gamma}\,, \quad H = \frac{1}{\gamma t}\,. \tag{96}$$

At the end of this subsection we also discuss the case of de Sitter solutions ($H =$ const.). Eliminating the $\omega\dot{\phi}^2$ term from Eqs. (94) and (95), we find that in order to get the solution (96), we must have

$$c_1\dot{\xi} = \alpha t\,, \tag{97}$$

where α is a constant. To realize a positive energy density ρ_c for $t > 0$ we require α is positive, which is assumed hereafter. The relation between α and γ is given by

$$\gamma = \frac{2\alpha + 3 \pm \sqrt{4\alpha^2 - 28\alpha + 9}}{2}\,. \tag{98}$$

An inflationary solution is obtained for $\gamma < 1$. We thus choose the lower sign in (98) and get the constraint on α:

$$0 < \alpha < 1/4\,. \tag{99}$$

It follows from Eqs. (94), (97) and (98) that

$$\rho_c = \frac{6(3-\gamma)}{5-\gamma}H^2 > 3H^2\,. \tag{100}$$

This implies that in order for Eq. (94) to have an inflationary solution we must have $\omega < 0$, i.e., a phantom-type scalar field. For simplicity, we choose $\omega = -1$. But this does not necessarily mean that the weak energy condition is violated when the correction term $\mathcal{L}_c$ is present. In fact from Eq. (95) or (96), we find $\dot{H} = -1/(\gamma t^2) < 0$ with $0 < \gamma < 1$. Taking the positive sign of $\dot{\phi}$, we obtain

$$\phi = \frac{1}{\gamma}\sqrt{6\left(\frac{4\alpha}{\gamma} - 1\right)}\,\ln t\,, \tag{101}$$

$$\xi(\phi) = \frac{\alpha}{2c_1}\exp\left(\frac{2\gamma\phi}{\sqrt{6(4\alpha/\gamma - 1)}}\right)\,. \tag{102}$$

Let us next derive the spectral indices of scalar and tensor perturbations. From Eq. (14) we find

$$c_{\mathcal{R}}^2 = \frac{(2-\gamma)(7-6\gamma+\gamma^2)}{5-4\gamma+\gamma^2}\,, \tag{103}$$

which is positive for $0 < \gamma < 1$. Since $\epsilon_1 = -\epsilon_2 = \gamma$, $\epsilon_3 = 0$ and $\epsilon_4 = 0$ in Eq. (28), we obtain

$$n_{\mathcal{R}} - 1 = 3 - \left|\frac{3-\gamma}{1-\gamma}\right|\,. \tag{104}$$

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[2] Recently there are a number of works which aim to explain late-time acceleration with the presence of the potential $V(\phi)$ [36]. For example, in the case of an exponential potential, a de Sitter fixed point appears because of the effect of the GB term [37].

When $0 < \gamma \ll 1$ this reduces to $n_{\mathcal{R}} \simeq 1 - 2\gamma$, thereby giving a nearly scale-invariant curvature perturbation.

Meanwhile the variable c_T^2 for the tensor perturbation is given by

$$c_T^2 = 2\gamma - 5\,, \tag{105}$$

which is negative for $0 < \gamma < 1$. This leads to strong negative instabilities for small-scale tensor perturbations invalidating the assumption of linear perturbations. Note that this type of instabilities has been also found in Ref. [38] (see also Ref. [39]). Unless the system is initially in a Minkowski vacuum state (in which c_T^2 is positive) before the GB term becomes important, a problem arises when we quantize tensor modes. It is interesting to note that the graviton is subject to this severe ultraviolet instability when we impose the condition to realize inflation.

Instead of the power-law solution (96), one may search for de Sitter solutions where H is a constant. From Eqs. (94) and (95), we get

$$\dot{\xi} = \frac{3}{10c_1 H} + Ae^{-5Ht}\,, \tag{106}$$

$$\omega\dot{\phi}^2 = -6H^2\left(1/5 + 4c_1 HAe^{-5Ht}\right)\,, \tag{107}$$

where A is an integration constant. Considering the asymptotic regime $e^{-5Ht} \to 0$, ω is required to be negative. We also find

$$c_{\mathcal{R}}^2 = 14/5\,, \quad c_T^2 = -5\,. \tag{108}$$

These are obtained by taking the limit $\gamma \to 0$ in Eqs. (103) and (105). Thus the de Sitter solution can be regarded as the special case of the power-law solution corresponding to the limit $\gamma \to 0$.

B. Models with $F(\phi) \sim e^{\mu\phi}$

We now consider the case in which one of the exponential terms dominates in the function $F(\phi)$, i.e., $F(\phi) = e^{\mu\phi}$. For the tree-level action with $F(\phi) = -\omega(\phi) = e^{-\phi}$ and $\xi(\phi) = \lambda e^{-\phi}$, it is known that there exists a de Sitter solution characterized by constant H and $\dot{\phi}$ [22, 23]. In what follows, we search for de Sitter solutions for the more general coupling $F(\phi) = e^{\mu\phi}$ and evaluate the spectral indices of scalar and tensor perturbations.

Since H^2 is a constant in Eq. (4), it is natural to find solutions in which each term on the r.h.s. of Eq. (4) is also a constant. This then requires that $\dot{\phi}$ should be a constant and that $\omega(\phi), \xi(\phi)$ have the same dependence as $F(\phi)$, i.e., $\omega(\phi) = \omega_0 e^{\mu\phi}$ and $\xi(\phi) = \lambda e^{\mu\phi}$. Hence from Eqs. (4) and (5) we obtain the following equations:

$$-\omega_0\dot{\phi}^2 + \mu H\dot{\phi} - \mu^2\dot{\phi}^2 + 2\lambda\left[2c_1\mu H^2\dot{\phi}(\mu\dot{\phi} - H) + c_2\dot{\phi}^4\right] = 0\,, \tag{109}$$

$$6H^2 - \omega_0\dot{\phi}^2 + 6H\mu\dot{\phi} - 3\lambda(8c_1\mu H^3\dot{\phi} - c_2\dot{\phi}^4) = 0\,. \tag{110}$$

These have a trivial solution $(\dot{\phi}, H) = (0, 0)$. However there exist a number of de Sitter fixed points with constant $\dot{\phi}$ and H depending on the model parameters $\omega_0, \lambda, c_1, c_2$ and μ. In this case, we have $\epsilon_1 = \epsilon_2 = 0$, $\epsilon_3 = \epsilon_4/2 = \epsilon_6 = \mu\dot{\phi}/2H$ in Eq. (28). Moreover since $c_{\mathcal{R}}^2$ and c_T^2 are constants, one can use the formula given in Eqs. (32), (34) and (27), namely

$$n_{\mathcal{R}} - 1 = n_T = 3 - \left|3 + \frac{\mu\dot{\phi}}{H}\right|\,, \tag{111}$$

$$r = 8\frac{\omega\dot{\phi}^2 + \frac{3(\dot{F}+Q_1)^2}{2F+Q_2} + Q_3}{\left(H + \frac{\dot{F}+Q_1}{2F+Q_2}\right)^2\left(F + \frac{Q_2}{2}\right)}\left(\frac{c_{\mathcal{R}}^2}{c_T^2}\right)^{\nu_{\mathcal{R}}}\,, \tag{112}$$

where we have used $\nu_{\mathcal{R}} = \nu_T = |\epsilon_3 + 3/2|$. This result is valid if both $c_{\mathcal{R}}^2$ and c_T^2 are positive. If either $c_{\mathcal{R}}^2$ or c_T^2 is negative, we confront with ultraviolet instabilities for small-scale perturbations. Note also that the tensor-to-scalar ratio becomes complex. Equation (111) shows that when $|\mu\dot{\phi}/H| \ll 1$ one can obtain nearly scale-invariant spectra of curvature perturbations.

At the tree level, one has $\mu = -1$, $\omega_0 = -1$, $\lambda = -1/4$ (for bosonic strings), $c_1 = 1$ and $c_2 = -1$. Then we get the fixed point $(\dot{\phi}, H) = (1.404, 0.616)$, which agrees with the result in Ref. [22]. In this case the spectral indices of scalar and tensor perturbations are highly blue-tilted, i.e., $n_{\mathcal{R}} - 1 = n_T = 2.28$. Moreover we have $c_{\mathcal{R}}^2 = -7.56$ whereas $c_T^2 = 22.2$, which means that the scalar perturbation exhibits violent negative instabilities on small scales.

In Table I we show de Sitter fixed points $(\dot{\phi}, H)$ together with $n_{\mathcal{R}}$, n_T, $c_{\mathcal{R}}^2$, c_T^2 and r for a number of different model parameters. Since Eqs. (109) and (110) are invariant under the simultaneous sign changes of μ and $\dot{\phi}$, it is sufficient to investigate the case of positive μ. Generally if we choose the values of μ larger than order unity, the spectral indices deviate from scale-invariant ones and are incompatible with observations. When $\mu \ll 1$ it is possible to get $n_{\mathcal{R}} \simeq 1$ because $|\mu\dot{\phi}|$ can be much smaller than H. However in such a situation the quantity c_T^2 typically becomes negative as we see in the cases (c), (d) and (i) in Table I. This is consistent with the results of constant $F(\phi)$ obtained in the previous subsection.

There exist exceptional situations which lead to $n_{\mathcal{R}} - 1 \simeq n_T \simeq 0$ and $r \ll 1$ with positive $c_{\mathcal{R}}^2$ and c_T^2. Such an example is given by the fixed point (f2) in Table I. We find that this is similar to the case in which the GB term is absent, i.e., $c_1 = 0$ and $c_2 = -1$. In fact, when $\lambda = 1/4$, $c_1 = 0$, $c_2 = -1$ and $\mu = 10^{-2}$, we have $(\dot{\phi}, H) = (1.4162, 0.4035)$, $n_{\mathcal{R}} = 0.9649$, $n_T = -0.0351$, $c_{\mathcal{R}}^2 = 1.44 \times 10^{-3}$, $c_T^2 = 1$ and $r = 9.3 \times 10^{-3}$, whose values are similar to those given in the case (f2). Hence the model (f2) is not much different from the kinetic inflation we discussed in the previous section. Provided that the GB term is subdominant relative to the $c_2(\nabla\phi)^4$ term, it is possible to realize the observationally supported density perturbation with the suppressed tensor-to-scalar ra-

Name	Parameters	$(\dot\phi, H)$	$n_{\mathcal{R}}$	n_T	$c_{\mathcal{R}}^2$	c_T^2	r
(a)	$\lambda=-1/4, c_1=1, c_2=-1, \mu=1$	$(-1.40, 0.62)$	3.28	2.28	-7.56	22.2	–
(b)	$\lambda=-1/4, c_1=1, c_2=-1, \mu=10^{-2}$	$(-5.94, 22.3)$	1.0027	2.7×10^{-3}	1.72	-3.10	–
(c)	$\lambda=-1/4, c_1=1, c_2=0, \mu\geq1$	–	–	–	–	–	–
(d1)	$\lambda=-1/4, c_1=1, c_2=0, \mu=10^{-2}$	$(-4.56\times10^4, 91.3)$	2.00	1.00	1.40	-5.00	–
(d2)	$\lambda=-1/4, c_1=1, c_2=0, \mu=10^{-2}$	$(-11.5, 10.4)$	1.011	0.011	2.89	-5.29	–
(e1)	$\lambda=1/4, c_1=1, c_2=-1, \mu=1$	$(1.22, 1.48)$	0.174	-0.826	2.32	0.611	46.4
(e2)	$\lambda=1/4, c_1=1, c_2=-1, \mu=1$	$(1.46, 1.35)$	-0.079	-1.079	-0.819	1.162	–
(f1)	$\lambda=1/4, c_1=1, c_2=-1, \mu=10^{-2}$	$(-1.41, 0.41)$	1.034	0.034	-1.2×10^{-3}	0.994	–
(f2)	$\lambda=1/4, c_1=1, c_2=-1, \mu=10^{-2}$	$(1.4158, 0.4042)$	0.9650	-0.0350	1.20×10^{-3}	1.006	7.1×10^{-3}
(g1)	$\lambda=1/4, c_1=1, c_2=-1, \mu=10$	$(-1.95, 1.00)$	-12.4	-13.4	0.19	-18.4	–
(g2)	$\lambda=1/4, c_1=1, c_2=-1, \mu=10$	$(1.84, 1.00)$	-17.4	-18.4	0.17	19.3	–
(g3)	$\lambda=1/4, c_1=1, c_2=-1, \mu=10$	$(0.14, 1.42)$	9.7×10^{-3}	-0.99	1.007	0.98	5.73
(h)	$\lambda=1/4, c_1=1, c_2=0, \mu=1$	$(1.03, 1.64)$	0.37	-0.63	1.26	0.095	1.15×10^3
(i)	$\lambda=1/4, c_1=1, c_2=0, \mu=10^{-2}$	$(11.5, 10.5)$	0.99	-0.01	2.72	-4.74	–
(j)	$\lambda=1/4, c_1=1, c_2=0, \mu=10$	$(0.14, 1.42)$	9.8×10^{-3}	-0.99	1.007	0.98	5.73

TABLE I: De-Sitter fixed points $(\dot\phi, H)$ for a number of different combinations of λ, c_1, c_2 and μ with $\omega_0=-1$. In the case (c) there exists no de Sitter fixed point. The tensor-to-scalar ratio becomes a complex number if either $c_{\mathcal{R}}^2$ or c_T^2 is negative.

tio. However if the GB term dominates over $c_2(\nabla\dot\phi)^4$, we are faced with the ultraviolet instability of tensor perturbations while nearly scale-invariant spectra of scalar perturbations are possible. This is a generic property associated with inflation induced by the GB term. Table I corresponds to the parameter $\omega_0=-1$, but we have also examined the case $\omega_0=1$. We have thus confirmed that it is difficult to satisfy all the conditions $n_{\mathcal{R}}\sim1$, $n_T\sim0$, $r\ll1$ and $c_{\mathcal{R}}^2, c_T^2>0$ if the correction term $\mathcal{L}_c$ is dominated by the GB term.

VII. CONCLUSIONS

In this paper we have discussed the possibility to obtain inflationary solutions and observationally supported density perturbations for the low-energy string effective action given in (1). The important quantities which are directly linked to observations are the spectral indices $n_{\mathcal{R}}$ and n_T together with the tensor-to-scalar ratio r. Provided that $c_{\mathcal{R}}^2$ and c_T^2 are positive constants in perturbation equations, we obtain the power spectra of scalar and tensor metric perturbations in Eqs. (19) and (25), respectively, together with the ratio r given in Eq. (27). The spectral indices of scalar and tensor perturbations are given by Eqs. (32) and (34), respectively. Note that these results are valid not only for slow-roll inflation ($|\epsilon_i|\ll1$) but also for the non slow-roll models with constant ϵ_i that often appear in dilaton gravity.

The action (1) is transformed to the Einstein frame action (40) by a conformal transformation (39). Since both the curvature perturbation $\mathcal{R}$ and the tensor perturbation h are invariant under the transformation in dilaton gravity, it is sometimes convenient to study perturbation spectra in the Einstein frame in order to confront with observations. For the models without both the dilaton potential $V(\phi)$ and the higher-order correction $\mathcal{L}_c$, it is not possible to obtain nearly scale-invariant spectra of density perturbations because the system is dominated by the kinetic energy of the field. Even in the presence of the dilaton potential, except for a specific case in which $V(\phi)$ is proportional to $F^2(\phi)$, it is required that the dilaton is effectively decoupled from gravity together with the existence of a slowly varying dilaton potential.

This situation changes if the second-order correction $\mathcal{L}_c$ given by Eq. (2) is present. When $c_1=0$, kinetic inflation is realized in the Einstein frame if the function $K(\phi)$ becomes negative. In fact this happens for two-derivative perturbative corrections to the Kahler potential in heterotic string theory, see Eq. (78). Along the de Sitter solution (76), the spectral index n_T and the tensor-to-scalar ratio r vanish whereas $n_{\mathcal{R}}-1$ does not. If the function $L(\phi)$ is constant in Eq. (74), we found that the spectrum of the curvature perturbation is blue-tilted ($n_{\mathcal{R}}>1$) in the case where $K(\phi)$ changes its sign from negative to positive. For general $L(\phi)$ the spectrum is either red- or blue-tilted, but $n_{\mathcal{R}}$ is very close to scale-invariant one if $K(\phi)$ and $L(\phi)$ are described by constants plus exponential terms. Moreover, for power-law solutions in which the scale factor is given by Eq. (90), the spectral index of the curvature perturbation is red-tilted and can be nearly scale-invariant, consistent with the WMAP3 data.

When the Gauss-Bonnet term is present in addition to the $(\nabla\phi)^4$ term, we have found a number of situations in which inflationary solutions are obtained. First we studied the case of constant $F(\phi)$ in the absence of the $(\nabla\phi)^4$ term and showed that power-law inflation is realized in such a case. Although scale-invariant spectra

are generated for curvature perturbations, tensor perturbations are faced with ultraviolet instabilities associated with negative coefficient c_T^2 when inflation occurs. We have also studied the case where the function $F(\phi)$ is given by $F(\phi) = e^{\mu\phi}$ and showed the existence of de Sitter solutions characterized by constant H and $\dot{\phi}$. If μ is larger than order one, the spectra of density perturbations deviate from scale-invariant ones. When $\mu \ll 1$ it is possible to generate nearly scale-invariant curvature perturbations, but tensor perturbations again suffer from negative instabilities on small scales if the correction $\mathcal{L}_c$ is dominated by the Gauss-Bonnet term. However as long as the $(\nabla\phi)^4$ term dominates over the Gauss-Bonnet correction, one can avoid this problem as it happens in kinetic inflation.

The results in our paper tell us that the condition for inflation, $|\dot{H}/H^2| \ll 1$, is not enough to generate nearly scale-invariant spectra of density perturbations. In addition we generally require that the parameters ϵ_i defined in Eq. (28) are smaller than order unity and also need to check the signs of $c_{\mathcal{R}}^2$ and c_T^2 to avoid ultraviolet instabilities. We have shown that these conditions are satisfied for some of the models as in the kinetic inflation discussed in Sec. V. Other models with higher-order corrections, can also satisfy these conditions, provided that the GB term is not dominant. It is of interest to extend our analysis to more general string models as given, e.g., in the works [40].

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